

Teaching Mathematics with Scientific Notebook

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A broad range of new computer packages have been developed in recent years, which are intended to help in teaching subjects in various areas, in particular, in teaching mathematics. For most of teachers, these novel teaching environments are very attractive. With all their advantages, an unwanted feature is also common among them: they bring a lot of confusion in many respects vital to teaching. Therefore, an essential question arises: how do we really should teach mathematics using these new tools if they are confusing, while being so powerful in solving any mathematical problem? Another question arises: what will the student learn if the program is capable of doing anything and everything for him or her?

In the paper, some of the major traps are pin-pointed that teachers may encounter in using mathematical software for teaching. The major instances where a computer program may expand both our teaching abilities and teaching environment are also identified. In closing, the paper shows as well how the teacher and student alike may benefit from using mathematical packages for teaching.

Introduction

New technology is always exciting and many people jump into it just for the pleasure of exploring something new. This concerns also various computer packages for teaching mathematics. I witnessed a number of presentations where somebody was showing off the various features of a marvelous program. Very often, too often in my opinion, I heard the words “look how easy this is, click and the result is ready”. So, click and an integral is calculated, click and an equation is solved, and so on. In such situations I would like to paraphrase these words into “click, you do not even need to think”.

This easy way of doing mathematics is confusing for people who have to teach mathematics. How can we teach some concepts, if the program performs the whole operation? How can we teach finding the greatest common divisor, partial fractions, differentiation or integration if just one click is enough to get the answer? Where is the learning process?

I find also that some teachers fall into the trap of fascination with a new program and instead of teaching mathematical concepts they rather teach how to master the program. Another, not very common but nonetheless observed case, is when the teacher gives the students a number of basic examples and asks them to solve one by one using a computer.

This way, learning mathematical concepts is replaced by a kind of pseudo-computing knowledge. Many people forget that while teaching mathematics, we do not teach only how to calculate something, we teach much more. Let me remind you about the major aspects of teaching and learning of mathematics.

1. Mathematics is a part of our history. Mathematical concepts were developed by real people and are located in real time. Very often such concepts are a part of a historical event.

2. Mathematics is also a part of our World culture. It contains the input of many nations: Greek, Arab, Indian, Polish, etc. Mathematical patterns are used in architecture, art and in many other forms of design.
3. While learning mathematics we learn problem solving techniques. For young generations learning mathematics is a kind of brain-jogging; it is a healthy activity for the brain just like jogging is for our body.
4. Finally, we learn mathematical concepts that we will use in everyday life and work, in many areas of science and engineering.

These, and perhaps many more arguments, show how important mathematics is to our society. This brings up the question - how shall we utilize the new computer techniques in order not to lose anything from the above list and perhaps even gain something?

Some important expectations

Allow me to analyze a computer program for teaching mathematics and to point out some of the features that, I think, can be useful for educators. As an example here, I will use my favorite tool *Scientific Notebook*, which in my opinion is the richest tool for teaching mathematics. However, many features that I describe here can be found in a number of other packages - *Derive*, *Mathematica*, *Maple*, *LiveMath*, *Tangible MATH*, etc. Below, I enclose the list of the most important features that I think a computer program for teaching mathematics should have. Thus, in my opinion, such a program should help us to:

- Prepare and publish teaching materials
- Increase our computational power
- Explore and experiment with mathematical concepts
- Obtain graphs of mathematical objects
- Improve communication between teacher and student
- Display concepts online and support distance education
- Test and train students

You can find all of these features and much more in *Scientific Notebook*.

Preparation and publishing teaching materials

The common problem in writing mathematical textbooks, tests and articles, is the way of inputting mathematical formulae. All popular word processing tools are acceptable if we have a document with only a few formulae. However, when we have to write a large document with hundreds of formulae, *Scientific Notebook* is invaluable. In this program, typing in a formula is easy, and in most cases we do not need to go through a maze of toolbars and toolboxes. We type most of the formulae in much the same way as we would type in regular text. Formulae are a part of the text, they are not embedded objects, and for this reason the program doesn't end up slowing down on bulky documents.

Scientific Notebook uses LaTeX for formatting documents and formulae. This means even very large documents are small files. Finally, *Scientific Notebook* produces graphs of mathematical objects in WMF format, which is a vector format. This means our graphs have always the same quality and it doesn't matter if we stretch or resize them.

Recently I presented *Scientific Notebook* to a group of Chinese mathematics teachers. Even in the first ten minutes of my presentation, many of them commented *Scientific Notebook* as the best tool for editing mathematics they had ever seen. This appreciation is very important. We all know how difficult editing English text with mathematical formulae is, now you can imagine doing the same in Chinese where each character is built out of a few elements.

Lecture notes prepared in *Scientific Notebook* can be printed as well as used for online reading. In this last case we obtain an interactive electronic book that the student can read while at the same time performing calculations, solving questions, adding graphs, etc. This makes such electronic books very attractive for students. It is worth mentioning the names of some mathematicians that have already written such electronic books: Darel Hardy, Carol Walker, Jonathan Lewin, Fred Szabo, and many others (see [1],[4],[5],[8]).

Expanding computational power

Scientific Notebook comes bundled with the computer algebra system *Maple* for which it acts as a friendly interface. All mathematical symbolism in a *Scientific Notebook* document is understood by Maple exactly as we would understand that symbolism if it were written with pencil on paper. We can therefore perform many complicated computing functions with ease. Thus, for example, if we point at the expression $\int e^{ax} \cos bx \, dx$ and click on the Evaluate button in the toolbars, we obtain

$$\int e^{ax} \cos bx \, dx = \frac{a}{a^2 + b^2} e^{ax} \cos bx + \frac{b}{a^2 + b^2} e^{ax} \sin bx$$

I am sure that we should not use mathematical packages for solving fundamental problems, unless we are asking students to check the results of their work. Thus the problems: find the greatest common divisor, find the limit $\lim_{x \rightarrow 0} \frac{\sin x}{x}$, evaluate the integral $\int \sin x \cos x \, dx$, differentiate the function $(\sin x)^{\cos x}$, etc., should be solved by the student manually. Such problems are aimed to teach students the techniques of performing some basic mathematical operations. All such problems can be solved by a computer program in just one click, while at the same time the student solving them manually has to perform a number of various operations. Such operations can follow some special patterns, like integrating by parts, differentiating logarithmic functions etc. These patterns are a very important part of mathematical education. We must think twice before removing them from the mathematical curriculum. Another important thing in such examples is the use of some unique solving techniques (e.g. partial fractions, Euclid's algorithm, etc.), that we can successfully apply in some other mathematical constructions.

Textbooks for high school mathematics as well as university mathematics contain a number of problems where it is important to find a way to solve the problem. In these cases a particular calculation used while solving the problem can be quite unimportant and, if possible, could be avoided. This, is the place where we can successfully bypass most of the time consuming calculations and concentrate on the idea of the solution, by applying a computer program. In such cases *Scientific Notebook* is invaluable. Its natural and easy to use interface as well as the powerful Maple computing kernel, remove from us the whole problem of inputting mathematics and performing calculations. We can then concentrate on solving the problem.

It is very important to note that using *Scientific Notebook* we do not need to learn any special programming language for programming mathematical operations. In these cases the “just the click” method is quite useful.

Here is a typical high school problem (see [2]), that takes some time while solving it manually, while doing it with *Scientific Notebook* takes a few minutes without losing anything important.

LOAN PROBLEM: A bank loan of \$500 is arranged to be repaid in two years by equal monthly installments. Interest, calculated monthly, is charged at 11% p.a. on the remaining debt. Calculate the monthly repayment if the first repayment is to be made one month after the loan is granted.

Exploring and experimenting with mathematical concepts

The above LOAN problem is not only time consuming if we solve it on paper. There is much missing in the paper solution. Imagine that students, as well as the teacher, want to check the same problem, but for a different interest rate - say 11.3% - or, that some additional payments are made as time goes on. In this situation the computer solution can be easily modified and we can consider a number of interesting cases. In each case we omit the time consuming calculations and we concentrate only on changing parameters and analyzing the obtained results.

While solving mathematical problems, we often have to analyze our results on a graph. With the use of a computer program it is very easy to change the coefficients of the function, domain and range, to find out how the graph changes. This way we can easily analyze not a single case but a number of cases.

A typical teacher's question - “check what will happen if” is very easy to deal with if we have a computer and a good mathematical program.

A number of other computer programs can be used for experimenting with mathematics. One of them, *MathView* (recently changed its owner and name to *LiveMath*), is a well known package for teaching high school mathematics.

Visualization of mathematical objects

Graphs are the essence of many mathematical considerations. In many situations a good graph is much better than formal proof. On the other hand we all know how difficult is to make a good graph on a whiteboard or even on paper. Such graphs, even the best, are quite static and every time when we wish to change parameters of the graphed object we have to make the graph again. With the use of mathematical software it is very easy to plot a graph and experiment with it. For instance, using *Scientific Notebook* we can plot 2D and 3D graphs, rotate and move them, change the accuracy or range of the variables.

We are not limited to graphs of functions. For instance, while discussing the integral of a function, the teacher can use *Scientific Notebook* to produce graphs of the function and its approximate integrals. Then he can show how, while changing the number of points, he changes the area covered by rectangles. Finally, he can calculate the approximate integral for various numbers of interpolating rectangles.

Example: Visualization of the approximate integral $\int_0^{10} \frac{1}{x^2+1} dx$.

We can obtain an approximate integral for both cases:

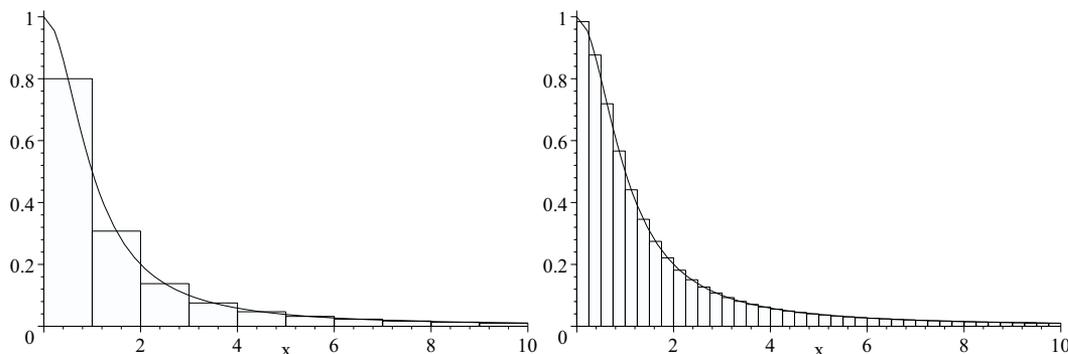


Figure 5: Interpolation of $\int_0^{10} \frac{1}{x^2+1} dx$ with 10 and 40 intervals.

$$\begin{aligned} \int_0^{10} \frac{1}{x^2+1} dx &\approx \sum_{i=0}^9 \frac{1}{i^2+1} = \frac{33\,054\,527}{16\,762\,850} = 1.9719 \\ \int_0^{10} \frac{1}{x^2+1} dx &\approx \sum_{i=0}^{39} \frac{1}{\left(\frac{1}{4}i\right)^2+1} \\ &= \frac{150\,118\,109\,347\,395\,797\,020\,028\,120\,011\,499\,883\,587\,233\,000\,599}{94\,125\,027\,820\,081\,777\,559\,063\,695\,169\,885\,530\,330\,766\,813\,000} \\ &= 1.5949 \end{aligned}$$

or calculate the approximate integral using the midpoint rule for 100 or more intervals.

$$\int_0^{10} \frac{1}{x^2+1} dx = \frac{1}{10} \sum_{i=0}^{99} \frac{1}{\left(\frac{1}{10}i + \frac{1}{20}\right)^2 + 1} = 1.4711$$

This, and hundreds of other examples, show how useful the graphing features of *Scientific Notebook* can be.

Improving communication between teachers and students

Communication between us, teachers, and students is not only talking and listening. We have to deliver lectures, assignments, and tests to our students as well as receive results from them. This can be done in many ways—on paper, whiteboard or blackboard. *Scientific Notebook* offers mathematics teachers a more interesting way. I know many university professors that are using *Scientific Notebook* as a kind of electronic whiteboard. One of them is Prof. Jonathan Lewin from Kennesaw State University in USA. Here is how he describes his teaching with the use of *Scientific Notebook*:

“I no longer use a blackboard and, in fact, I have not touched a piece of chalk for several years. I teach in a lecture room that is equipped with an LCD projector (a Proxima 5800) and a connection to the Kennesaw State University network that connects me to the internet by a T1 line. I carry my laptop computer into the classroom with me. Before the class begins I connect my laptop to the projector and to the campus network. In addition, I connect the laptop to a conventional mouse and a Microsoft Natural Elite keyboard. I sit at the front of the room with my laptop facing my students. I look at the display on my computer and the students see the same display on the projection screen behind me.” You can find more information about Prof. Lewin’s teaching technique on his web site <http://science.kennesaw.edu/~jlewin/>

For about two years, I was using *Scientific Notebook* while teaching Discrete Mathematics to Papua New Guinea students, though in slightly different conditions (see [6],[7]). My students appreciated this way of lecturing very much. Usually, immediately after my lecture I would save a rough version of my notes on the network. Later on, I used to polish these notes, eventually giving the final file to the students or placing it on a WWW or FTP server where they could access it.

On the other side, my students could do exactly the same. They could do their work in *Scientific Notebook* and send it to me by e-mail; or, using FTP, simply save their work on my server.

There is another important advantage of using *Scientific Notebook* by students for writing assignments and other works. They improve their writing skills and learn how to properly formulate mathematical concepts.

Presenting concepts online and supporting distance education

From teaching mathematics with *Scientific Notebook* as an electronic whiteboard or producing electronic textbooks, to publishing on the Internet, is just one small step. Lately, many people talk about the opportunities of using the Internet in teaching. But not many of them know how easy it is to make a mathematical web site with the use of *Scientific Notebook*.

While editing documents in *Scientific Notebook* we can easily add links to various places in the document, or to other documents on a local machine, or even to documents located somewhere else on the Internet. This way, *Scientific Notebook* serves as an editor of mathematical web pages. On the receiving end, the same *Scientific Notebook* works as a web browser displaying well formatted, colorful mathematical documents with graphs, sound and video. There is a great advantage of such web pages. Students may save such a page on their own computer and use it as a starting point for their own mathematical investigations. They may recalculate the displayed results, experiment with them, re-plot graphs, add their own comments and, if necessary, pass the page on to anybody, including their teacher.

Scientific Notebook is a great tool for various distance education centers. At the moment, some important universities are using *Scientific Notebook* in their distance education centers. Perhaps the most famous distance education project using *Scientific Notebook* is run by Prof. Szabo at the Concordia University. This is a complete web learning initiative run in distance education mode. *Scientific Notebook* is the main package used as an electronic whiteboard, but students also have access to *Maple V* and *Mathematica*. The whole project is run on a specialized web server WebCD that provides not only web space but also other facilities: e-mail, chat, etc.

Another important project run with *Scientific Notebook* is the WebCalc project - Interactive, Multimedia Calculus, run by the WebCalc Group (Don Allen, Maury Rahe, Mike Stecher and Phil Yasskin) at the Texas A&M University. This is an advanced, totally web-based project, that in September 1999 will be run strictly in a distance mode. You may find more information about this project at <http://www.math.tamu.edu/~webcalc/> or, if you have *Scientific Notebook* on your computer you may jump directly to the WebCalc course at WebCalc web site (see [1]).

Recently one of the biggest distance education universities, the Hong Kong Open University, started using *Scientific Notebook* for teaching a number of their courses. I have noticed also that some other universities have mathematical web sites with a limited selection of courses displayed in *Scientific Notebook* format.

Scientific Notebook is not the only mathematical program that produces and displays mathematical web pages. We can obtain a similar result using some of the other packages. For instance *Maple V R5* produces two kinds of web-ready documents. One of them is a typical HTML format with formulae embedded as GIF images and graphs also presented in GIF format. Another, more exciting, feature of *Maple V* is the possibility of saving a *Maple* worksheet in *MapleExplorer* format. This is a TeX file like in *Scientific Notebook*. Unfortunately, Waterloo Maple recently stopped distributing *MapleExplorer* and this way of displaying *Maple* mathematical web pages is now closed.

The queen of mathematical software, *Mathematica*, also produces web pages in HTML format and formulae as GIF files. Using *Mathematica*, we may also obtain HTML pages with formulae coded in MathML. Such web pages can be displayed by *Internet Explorer* or *Netscape* with the *IBM Techeplorer* plug-in. However, *Scientific Notebook* documents on the web offer much more than just a way of displaying mathematical contents. These are live mathematical documents. Live in the sense that after saving them on a local computer we can still continue the calculations published on these pages.

Testing and training students

Teachers spend a significant amount of their time marking students' assignments, tests and exams. There have been some attempts to make this process faster and less boring. In recent years a few computer programs were developed for testing students. However, at the moment, only *Scientific Notebook* offers unique tools for developing mathematical tests and grading students by a computer-local, or on the web server.

Using *Scientific Notebook* we can write a test file and program in it many elements that can be generated by the program randomly. The *Scientific Notebook* Exam Builder will compile such a file into a test file choosing random values for some elements. This means each time when a student takes such test on his computer, he gets completely new data. For example, we can define constants: $a := \text{rand}(100)$, $b := \text{rand}(100)$, $c := \text{rand}(100)$ and the function $f := ae^b \sin cx$. While taking such test online, the student may obtain functions like $f(x) = 3e^4 \sin 5x$, $f(x) = 31e^{89} \sin 78x$, $f(x) = 48e^{44} \sin 9x$, $f(x) = 4e^5 \sin 8x$, etc. This feature makes *Scientific Notebook* tests also a good training tool. While teaching Papua New Guinea students, I used to give the students the test file a week before the final test. During this week they would do this test again and again, each time with different data. This way they learned much more than just reading a textbook or solving questions from a textbook.

Using the recent version of *Scientific Notebook* we can generate test files on the web server and record students' results on the web server. This is another feature that can be very useful for distance education mathematical web site developers.

Conclusions

Recently I made a number of workshops for Chinese teachers of mathematics. I presented to them *Scientific Notebook* and its unique features. There were three themes: introduction to teaching mathematics with *Scientific Notebook*, development of the web site with *Scientific Notebook*, and computer assessment in mathematics. As a result of these workshops a few initiatives were taken. Two of them are worth mentioning here: translation of the *Canotta's Mathematics Test Bank* (see [3]) into an electronic form and the development of a web site for teaching algebra in a Chinese high school. The last project is the theme of the M.Sc. thesis of

my student from Inter-University Institute of Macau.

My research on using various mathematical packages in mathematics education shows that we can use them in many different ways and enhance the traditional textbook & whiteboard methods of teaching mathematics. We can go much further than only using them in classroom teaching. Internet and distance education can benefit from such computer programs. Teachers after a short series of workshops very easily grasp the concept of teaching mathematics with a computer and spontaneously develop ideas for many interesting projects. Perhaps it is also worthwhile to think about a kind of mathematical olympiad - *Mathematics with Technology*, and writing a series of textbooks with sets of problems that can be solved on the computer rather than on paper.

References

- [1] Allen D., Rahe, M., Stecher, M. & Yasskin, P., (1998). *A First Course in Calculus: A Web Based, Interactive, Multimedia Calculus*, Texas A&M University, electronic publication at <http://www.math.tamu.edu/~webcalc/mindex.tex>
- [2] Bostock L. & S. Chandler, (1994). *Core Maths for A-level*, Stanley Thornes Ltd, UK.
- [3] *Canotta's Mathematics Test Bank*(1998), Canotta Publishing Co., Ltd., Hong Kong.
- [4] Hardy, D.W. & Walker, C.L.,(1997). *Doing Calculus with Scientific Notebook*, Brooks/Cole Publishing Co., California. (Electronic publication distributed on CD with *Scientific Notebook*).
- [5] Lewin, J., (1997). *Precalculus with Scientific Notebook*, Kendall/Hunt Publishing Company, electronic publication at http://science.kennesaw.edu/~jlewin/precalculus/lewin_sn.tex
- [6] Majewski, M.,(1999). Experiences in Teaching Mathematics With Scientific Notebook, 8th *Southeast Asian Conference on Mathematics Education*, Manila.
- [7] Majewski M., (1998). Scientific Notebook - an Interactive Approach in Teaching Mathematics, *International Conference on the Teaching of Mathematics*, John Wiley & Sons, New York.
- [8] Szabo, M. E.,(1997). *Linear Algebra and Its Applications Illustrated in Scientific Notebook*, Concordia University. (Electronic publication distributed on CD with *Scientific Workplace*.)