Calculus Reform: Changing the Syllabus

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Student and staff dissatisfaction with the first-year Calculus Reform course at ADFA over the past few years has caused us to re-think our approach. We have re-designed the course to place greater emphasis on the modelling process, with differential equations pre-eminent. The need to solve differential equations leads to the introduction of most of the usual components of a Calculus course.

Introduction

First-year Mathematics at the Australian Defence Force Academy (ADFA) is taught at only one level. The enrolment is about 60% Engineering and 40% Science/Arts students. In recent years, both groups have had the same syllabus, but the methods of teaching have differed, as outlined below, because of the restrictions of the Engineering timetable and because of staffing constraints. Half the course is Calculus, with the other half made up of Linear Algebra (including Vectors and Complex Numbers), Probability and Discrete Dynamics. This paper is concerned with recent developments in the Calculus part of the course.

What were we doing?

Up until 1986 we used our own notes in a modified Keller plan, but this scheme proved to be too labour-intensive when our student numbers trebled when ADFA opened. We continued with our notes but with a standard assessment scheme (tests, exams) until 1988, when we adopted Larson and Hostetler [1] as a text because it provided a much more comprehensive and well illustrated set of notes than we could. In 1993, we changed our text to Thomas and Finney [2] and introduced graphics calculators, which were purchased by all students.

It rapidly became clear that the use of graphics calculators changed the way Calculus could be taught, and that the chosen text did not fit in well with how we wanted to structure the course. The Harvard Calculus Reform text [3] seemed to meet our criteria and was introduced in first year in 1994. Classes were still conducted as large lectures, but students were required to carry out some work in class.

In 1994, we also introduced a weekly two-hour lab for our Science/Arts first-year class. Students work in groups of four on course-related problems, handing in a group report at the end of the lab period. Other features of our first-year course introduced at this time were an intensive Catch-up Course held in the May break for struggling students, an Essential Maths Scheme and a Maths Study Centre, staffed two hours a day, where students can obtain help.

We continued using the Harvard text and a later edition [4] until the end of 1997 when, for reasons outlined in the next section and after extensive consultation with other members of the School (including an informal vote), we changed our text to Smith and Moore [5]. Another important change was dividing our Science/Arts Calculus class into groups of about 25, based initially on high-school results, each group with a staff member as instructor.

Although we all felt Smith and Moore was an excellent text (it won the vote), it became clear after using it for a few months that it too had serious shortcomings *for our students*.

Why weren't the courses successful for *our* students?

Most of our students come to first year with a reasonable background in Calculus, having done at least basic differentiation and integration. There is, however, a significant variation between the syllabuses in the different States and Territories, with the time our students devoted to Mathematics in Years 11 and 12 ranging from 200 hours to more than 400 hours.[1] We have students from all States and Territories, so that some of our first-year course has to be devoted to bringing all students up to a similar level in the areas which are continued in our first-year and subsequent courses.

What sorts of skills do our students arrive with? In the first week of semester, they all take an Essential Skills Test, which tests basic algebraic manipulation, differentiation and integration.[2] By and large, with a little revision, students can do these questions.

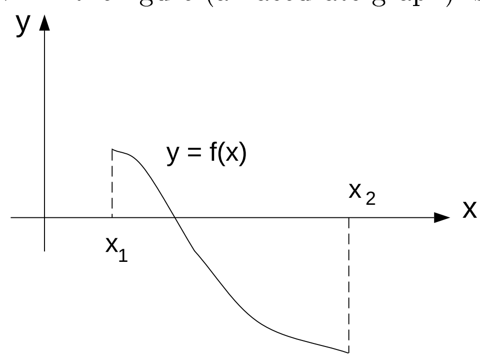
Students are also given what we call a Concepts Quiz, which aims to test their understanding of some of the concepts behind Calculus (function, the derivative, the definite integral, etc). One of the questions from this quiz is given below.

The integral $\int_{x_1}^{x_2} f(x)\, dx$ of the function f shown in the figure (an accurate graph) is

(a) positive

(b) negative

(c) either positive or negative, depending on the formula for f and the values of x_1 and x_2.



The results for the Concept Quiz are not nearly as good as those for the Essential Skills Test. Nearly half of the 25 students in the *top* Calculus group answered this question incorrectly. Clearly, the students' understanding of Calculus concepts and their ability to visualise them are well behind their ability to manipulate symbols according to certain rules [6]. Yet it is the understanding of the concepts that allows students to develop their Calculus skills and go on to more difficult areas such as differential equations and multi-variable Calculus.

It is this understanding that was emphasised in the early part of our course using the Harvard texts [3] and [4], with a considerable proportion of the first three chapters *A Library of Functions*, *The Derivative* and *The Definite Integral*, devoted to understanding the concepts and to incorporating the concepts into modelling-type problems. Students found the questions from

[1] Our current prerequisites (under review) are: *ACT* Major/minor; *NSW* 3-unit or high-level 2-unit; *Queensland* Maths B; *SA/NT* Maths 1 & 2; *Tasmania* Stage 2; *Victoria* 6 units, including Maths Methods 3 & 4, Specialist Maths 3 & 4; *WA* Geometry & Trigonometry, Calculus, Applicable Maths.

[2] Details of these tests can be found under *Courses* on the Mathematics and Statistics home page *http://www.ma.adfa.oz.au*.

this part of the course difficult, but, because they came under the heading *Function*, *Differentiation* or *Integration*, they thought they had covered the material at school and tried to use the methods learned there. This led to frustration, "*Why are you asking us strange questions on stuff we know?*", and a tendency to ignore the work in the hope their school-learned skills would carry them through in the end. This delusion of already knowing it all was easy to sustain amidst the pressures of their first semester at ADFA. Students waited until something obviously new came along before becoming serious about Maths. The good ones recovered and eventually did well,[3] but many of the weaker students struggled much more than they would have if they had taken the subject seriously from the beginning.

This student perception that they had done it all before and a worry about losing the good students were the principal reasons why we changed from the Harvard text [4] to Smith and Moore [5] in 1998. Smith and Moore introduces differential equations early on, giving most students something new.

Unfortunately, Smith and Moore assumes a relatively low level of prior Calculus knowledge: the basics are developed in the book, thoroughly but slowly. Much of the work in the book is done with little assumed knowledge of integration, which is only covered in the last chapter. We, the instructors, and the students, even the weaker ones, found this frustrating, as the students' knowledge of integration would have allowed them to solve many of the problems in a more straightforward, if less intuitive, way. An example is the method of separation of variables for first-order differential equations. We gave the students notes on this topic, but it was then difficult to blend this knowledge with the treatment in the book. Similar problems were found with differentiation. On the whole, the book did not go far enough in most topics, given the background of our students. While excellent to work through for its intended audience, the book was difficult to use when the order or depth of treatment of topics was changed (by us). Our students did not like the book principally for this reason.

At the end of 1998, we faced a dilemma. We liked the order in which topics were introduced in Smith and Moore, particularly the early introduction of differential equations, but the level and the approach were not suitable for our students. The Harvard text went into topics in a depth suitable for our students and was easier to use when we made modifications to the order of the topics, but students perceived most of the work in first semester as having been done at school.

Three topics seemed of particular importance and interest to us as applied mathematicians: differential equations, optimisation and integral problems set up using Riemann sums (volumes of solids, density problems, etc). All of these involve translating word problems into mathematical models, which then have to be solved. This is a skill which we seek to foster in first year. Understanding any new topic takes a reasonable exposure time, even when linked to previous knowledge; understanding differential equations, for example, something completely new to almost all students, takes even longer. Three or four weeks at the end of the course was certainly not sufficient. Students came away with little more than algorithms for solving some differential equations. Riemann-sum problems received only a short treatment too.

What did we change?

We resolved our dilemma in perhaps the obvious fashion by writing a fairly comprehensive set of notes for 1999, based on and to be used with the most recent version of the Harvard text

[3] The really good students did know it all already and often became bored with what seemed obvious to them.

[7]. The notes incorporate the basic theory, with references to the text for more detail, and contain many examples to be worked through with the class and exercises to be completed by the students in class. Spaces are left in the notes for students to write in their solutions, so that all their classwork ends up in one place. Solutions to all the exercises are available in a complete set of notes on the Web.

The motivation for all sections is problems of a modelling type, i.e. problems expressed in words, most of which (in first semester) require the setting up and solving of differential equations. The solution of first-order differential equations provides the vehicle for introducing other Calculus topics. For example, the method of separation of variables requires a study of integration, both algebraic and numerical. The use of slope fields and Euler's method necessitates a discussion of the derivative, and the velocity/time functions used to understand the derivative lead nicely into problems involving moving bodies, without and with air resistance.

The concept of a function is still covered first, but functions of two variables (new to all students) are also discussed. The study of particular functions, such as the exponential and log functions, is mostly left until they arise in solving a particular type of first-order differential equation. This means we start solving differential equations in about the fourth or fifth week of first semester, giving all students something new to study. The setting up and solving of word problems is also introduced early on, firstly to motivate the need to solve differential equations, and secondly to give the students a long exposure to this type of problem. Further word problems appear throughout the different sections, the complexity increasing as students become comfortable with the concept of a differential equation. The problems are graded, with the weaker students covering fewer problems than the better students. Harder problems involving differential equations and other topics in the course are given to students in the labs, where they work in groups and submit a group report.

An outline syllabus of our 1999 first-semester course is given below. All students own a graphics calculator (mostly a TI-83), which is an intrinsic part of the course.

- **Functions and Modelling:** linear, exponential and power functions; inverses; functions of several variables — representation by graphs, tables and formulas.

- **First-order Differential Equations:** what are differential equations, why study them and what do their solutions look like; separation of variables, which includes integration by substitution and using tables, numerical integration, the log and exponential functions; the derivative as a rate of change and as a slope — average and instantaneous rates of change, difference quotients and the derivative; slope fields, including equilibrium solutions and stability; Euler's method and the modified Euler's method; approximate algebraic solutions using Taylor polynomials and series.

Second semester followed a similar scheme of posing problems and then looking at the methods needed to solve them. Word problems provided the motivation for and the culmination of each topic.

- **Optimisation:** global maxima and minima; local maxima and minima, points of inflection.

- **Modelling with Second-Order Differential Equations:** periodic motion and trig functions; numerical solutions using Euler's method; algebraic solutions of linear homogeneous equations with constant coefficients; approximate algebraic solutions using Taylor series.

- **Modelling with Integration:** the basic ideas behind the definite integral using a velocity/distance problem; the Fundamental Theorem of Calculus (Euler's method vs Left Endpoint Rule); setting up integrals via Riemann sums, with applications to geometry, density problems, etc.
- **Introduction to Partial Differential Equations:** what they are, how they arise, examples; partial derivatives; a brief look at solutions to some common partial differential equations.

Conclusion

We, the instructors, are particularly happy with the course. It incorporates most of the elements of Calculus Reform, but allows us to tailor the topics to suit the background of our students and our personal judgements as to what is important.

Students find the course challenging, no-one has yet complained that they have done it all before, but there seems to be generally a high level of interest. The division of the class into smaller groups is also popular. The full extent of student opinion will not be known until we conduct an extensive survey in second semester.

All our first-year course material can be found under *Courses* on the Mathematics and Statistics home page at *http://www.ma.adfa.edu.oz.au.*

References

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[3] Hughes-Hallett, D., A.M. Gleason *et. al.*, (1992). *Calculus* (preliminary ed), John Wiley and Sons, New York.

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[5] Smith, D.A. & L.C. Moore, (1996). *Calculus Modeling and Application*, D.C. Heath, Lexington MA.

[6] Eisenberg, T. & T. Dreyfus, (1991). On the reluctance to visualize in Mathematics, *Visualization in Teaching and Learning Mathematics*, Mathematical Association of America, Washington, 25–37

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