Reform in Undergraduate Multivariable Calculus

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This paper reports on experiences in implementing a reform approach in second year multivariable calculus at Central Queensland University. Although research into teaching and learning in reform calculus is ongoing there has been little reported beyond first year undergraduate level. This trial compares the performance, using an identical syllabus and assessment scheme, of two differently prepared groups of students - one comprising distance students using an established traditional approach and the other comprising internal students using a reform approach.

Introduction

The reform calculus movement has grown from its beginnings in 1986 to the point where in 1995, according to an ongoing study sponsored by the American Science Foundation [1], over thirty percent of all students enrolled in calculus courses in America were learning the subject the reform way. Although research into aspects of teaching and learning in reform calculus is ongoing [2,3], including some limited reporting on the Australian scene [4,5], there is little mention of the reform approach beyond first year.

Reform calculus essentially involves thoroughly grounding students in the key concepts and applications of calculus with perhaps less emphasis on mastery of the attendant procedural skills via use of drill exercises. The subject may still be presented with rigour and use of drill but in a very different atmosphere than is often the case in more traditional settings. Students are introduced to fundamental concepts by considering problems and applications which are carefully set in context. An applications rich atmosphere is maintained and supported by using the so called "rule of three" whereby each aspect of the subject is presented and analysed from a graphical, numerical and algebraic viewpoint. Some practitioners extend this to the "rule of four" adding a verbal dimension whereby students are encouraged to talk about mathematical concepts as well as apply them.

By 1994 the reform approach had been extended beyond first year level to include multivariable calculus with implementations largely supported by a preliminary edition of the textbook commissioned by the Harvard consortium [6]. The publication, in 1997, of the first edition of this textbook and the subsequent publication of others in the reform and pseudo reform vein [7] has encouraged, at least in America, a more widespread acceptance of the approach. In the absence of reports in the literature it is not clear what interest there has been elsewhere but as of mid 1999 there are at least two Australian sites teaching multivariable calculus using a reform approach, these being the Australian Defence Force Academy (ADFA) since 1995 and Central Queensland University (CQU) since early 1999.

Implementation of reform multivariable calculus at CQU

A recent survey [5], found that although the reform approach, used for first year calculus subjects at CQU since 1995, did seem to improve pass rates and was popular with students there was no evidence to suggest that mathematical performance in subsequent undergraduate years had improved. This motivated a trial of the reform approach in a second year multivariable calculus subject where the performance, under an identical syllabus and assessment scheme, of reform prepared internal students was compared with that of more traditionally prepared distance students.

The trial ran over a twelve week term in the first half of 1999. Internal students used the Harvard reform textbook [6] whilst distance students used a more traditionally styled textbook [8]. Apart from some minor differences in the ordering of topics and some significant differences in emphasis both textbooks covered all the topics in the fairly typical multivariable calculus syllabus.

Although students were encouraged to use appropriate graphing software such use was not explicitly integrated into organised workshop sessions. This was partly due to time and resource constraints and partly due to difficulties in maintaining a consistent approach between internal and distance students.

Modes of Delivery

Internal (reform) sessions were allocated four hours each week. The original aim was to make each one hour session a mix of both theory and practice but this did not suit an already very tight timetable then further reduced to just eleven teaching weeks after taking into account public holidays. This meant that although the material was presented in the reform spirit much of it was delivered in a more traditional framework of three lectures dealing mostly with theory and one practical tutorial.

Distance (traditional) students relied mostly on their textbook [8] together with locally prepared accompanying resource materials which included a study guide providing a directed reading path through the textbook and suggesting appropriate exercises and activities.

Assessment

The assessment was based entirely on two assignments due in weeks six and ten respectively and on a three hour final examination. The assignments and examination paper were deliberately designed to include a mix of questions couched in both reform and more traditional styles. The reform style questions were taken from the resource materials which complemented the reform textbook and the traditional style questions were similar to simpler exercises posed in the distance students' textbook.

Evaluation of the Trial

Internal and distance student enrolments in the multivariable calculus unit since 1997 have rarely exceeded thirty making a formal statistical analysis of the trial results something of an overkill and any direct comparisons at best tentative. However the following relatively unsophisticated analysis, taken in conjunction with feedback in the form of an internal student survey to form a small pilot study, arguably does provide a plausible interpretation and may indicate more general trends.

Performance comparison

Results in first year calculus units offered at the university over the last ten years and including both pre and post reform data show a remarkably strong correlation between distance and overall internal student performance in both assignments and examinations. For example, averaged over the period 1990 to 1997, the mean final mark for both internal and distance students taking differential calculus subjects was 66.2% and 66.4% respectively.

Although the number of students taking second year multivariable calculus is much reduced the tight correlation evident above still seems to hold with the mean final mark for both internal and distance students over the period 1997 to 1998 being 66% and 68% respectively as shown in Table 1. It should be noted that this data includes passing students only as most failing students do not attempt all the assessment and in dropping out at different stages their inclusion skews the results markedly.

All other factors being equal, this close correlation is useful in assessing performance differences in 1999 between the internal reform students and the traditionally prepared distance students. Any significant variation is then a reasonable indicator of the effectiveness of one approach as compared with the other. As the results given in Table 2 show the 1999 trial results are remarkably consistent with those for 1997 and 1998 in that there has been little variation in performance within either the reform or traditionally prepared groups.

Mean mark	Internal students	Distance students
Assignments	77%	72%
Final Exam	61%	67%
Overall	66%	68%

Table 1. Pre trial, internal versus distance student results for 1997 and 1998

Mean mark	Internal reform group ($n = 10$)	Distance Trad. group ($n = 12$)
Assignments	73%	73%
Final Exam	60%	66%
Overall	64%	68%

Table 2. Reform trial results for Autumn semester 1999

A similar pattern holds for overall grade distributions. As Tables 3 and 4 show these have remained reasonably consistent for the traditionally prepared distance students over the period 1997 to 1999. A slight change, which might be attributable to the introduction of the reform approach, is evident in a 7% decrease in reform students gaining distinction grades in 1999.

Mark/100	Internal students	Distance students
$X > 75$	27%	38%
$45 < X < 75$	73%	62%

Table 3. Pre trial, breakdown of overall results for 1997 and 1998

Mark/100	Internal (reform) students	Distance (trad.) students
$X > 75$	20%	42%
$45 < X < 75$	80%	58%

Table 4. Reform trial breakdown of overall results for Autumn semester 1999

Factors affecting performance

The slightly better overall results consistently shown by distance students could, at least in part, be due to the fact that on average they are older and more mature than most internal students. This is noted, for example, by Robertson [9], who points out that distance students in mathematics subjects generally are almost always comparatively highly motivated and experienced.

Another factor affecting performance in the 1999 trial is that there are significant differences in emphasis between the two textbooks used. On first impressions these differences would seem likely to favour the reform students but there is evidence to suggest that the opposite could well be the case. In essence the reform textbook carefully emphasises concept development reinforced by examples and exercises using real world applications, whilst the traditional textbook puts less emphasis on explicit concept development, having relatively concise sections on theory followed by more demanding examples and exercises calling for relatively higher order algebraic and manipulative skills.

Discussing and interpreting performance differences

Given the extra emphasis on concept formation in the reform approach it might seem surprising that the reform group did not do better, especially considering that the two assignments and most of the final examination questions were taken directly from the resource materials supplied with the reform textbook and couched in a style and context more familiar to reform students than traditional ones. In fact the final examination results show that the traditionally prepared students in every case did as well and in many cases did better than the reform students on the reform style, specifically concept testing, questions. The only examination question that was answered marginally better by the reform group was one involving a typical Taylor approximation question where little more than a "plug and chug" approach was required.

Feedback from distance students indicates that their stronger overall and examination performance may be, at least in part, attributable to the very difficulties they must overcome in studying largely alone with a demanding traditional textbook. Distance students frequently complain about the difficulty in learning from a textbook which consistently demands a high level of algebraic skill and firm grasp of prerequisite mathematics. However many of them concede that it is this very difficulty that forces them to work hard to come to grips with the fundamental concepts and arguably to do so at least as effectively as the reform group. Furthermore the emphasis on non trivial algebra in examples and problems arguably hones their algebra and manipulation skills to a level beyond that of the reform prepared group. There is evidence to support this interpretation in the examination results which showed the traditional group scoring significantly higher marks for questions involving more involved algebra and manipulation. For example the two questions where the gap in favour of the traditional group was largest involved finding the gradient of a simple multivariable function in order to work out the equation of a tangent plane to a surface and the evaluation of a volume using a triple integral. Neither question was difficult at the conceptual level and both mainly involved implementing practiced procedures. The significant point is that, although most students demonstrated that they knew what needed to be done, the reform group showed significantly less mastery in tackling and correctly completing the relatively more demanding algebraic manipulations.

This brief discussion, whilst by no means conclusive, is getting to the essence of teaching and learning differences between the reform and more traditional approaches. In the writer's

opinion the important differences are not so much in how the content is presented but in how each approach aligns with typically very tight syllabuses in multivariable calculus. In this context time constraints seem to work more against the reform approach than a traditional one. The reform approach takes more time for basic concept formation and in a tight syllabus this must come at the expense of honing manipulation skills. In the same tight timetable a traditional approach, having less explicit emphasis on concept formation and focussing more on procedural and manipulative skills, encourages inquiring students to forge, link and maintain a concept base largely by themselves and in their own terms. In more traditional approaches and certainly the case of distance students this amounts to a largely self directed discovery learning process driven largely by necessity.

It might still seem odd that the traditionally prepared group handled all the examination questions which specifically tested concept assimilation better than the reform prepared group; especially given that the reform approach is supposedly more in tune with teaching and learning improvements espoused in modern constructivist theory. Perhaps this is explained by conceding that although concept formation and linkage are indeed central considerations in learning mathematics this does not preclude a concomitant emphasis on drill and mastery of procedure which have long been recognised as important parts of concept consolidation and integration [10].

In any event and irrespective of the author's interpretations a clear message coming from this trial is that the reform prepared students did not perform any better than the traditionally prepared ones.

Student survey results

A routine end of semester survey of internal students showed that they rated the reform approach highly. Their assessment on a seven point Likert scale (which ranged from Very Poor = 1, through Satisfactory = 4 to Outstanding = 7) was 5.3, 5.2 and 6.0 respectively for their assessment of the knowledge gained from the subject, enjoyment of the subject and learning gained from assignments. Repeating students in particular said that they found the reform approach provided a greater sense of coherence and concept linkage in what otherwise had often seemed a largely unconnected mix of procedures. The most frequent comment made by students was that too little time was allocated in a very crowded syllabus to digest the work properly and that much more tutorial time was desired.

Conclusion

The reform approach, implemented at any level in the undergraduate syllabus, is very demanding of resources. It is therefore a challenge, especially in an atmosphere of increasing resource constraints, to do it justice, especially as it was presented and supported by the textbook adopted [6]. To be fair, the compromises forced by a tight, traditionally structured timetable made this trial more of a 'pseudo reform' implementation. Nevertheless the reform spirit was pronounced making the approach very popular with students who appreciated the extra effort taken in concept formation and exploration.

An exit survey showed that most students seemed to agree that in learning the reform way they feel, in a relative sense, more confident, enthusiastic and informed. However there is no evidence that such positive feelings improve performance. In fact the traditionally prepared students did marginally better in overall grade distributions and markedly better in most of

the examination questions.

A judgment of this trial comes down to weighing the positive feelings that students have for the reform approach against its extra resource implications and a clear lack of performance benefits. In hindsight, given the time constraints involved, it would probably have been more appropriate to have adopted a more moderate reform approach and one better suited to local conditions. This could be attempted using, for example, a textbook which steers a more middle of the road course [7]. A further trial using such an adjusted approach is envisaged at CQU in early 2000.

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