Dealing with Diverse Student Backgrounds

Philip Swedosh
The University of Melbourne
Melbourne, Victoria
swedosh@ms.unimelb.edu.au

One of the greatest difficulties of teaching first year university mathematics is the diversity of backgrounds with which students enter. An important source of this diversity is the level to which students have mastered various basic concepts. A large proportion of students have mathematical misconceptions, which, unless something is done, persist. Eliminating these misconceptions is therefore crucial. A strategy based on Piaget's notion of cognitive conflict has successfully reduced the frequency of mathematical misconceptions exhibited by very bright tertiary students and that improvement persisted over time. This study investigates the effects of this method on average first year university students.

Introduction

Teaching first year university mathematics is associated with many difficulties, amongst which the diversity of backgrounds of entering students is one of the greatest. This difficulty arises largely due to the variability of levels to which students have mastered basic concepts. A surprisingly large proportion of students have mathematical misconceptions, which tend to persist if no action is taken. Several writers ([1]; [2]; [3]; [4]; [5]; [6]) have studied various aspects of mathematical misconceptions including their variety, frequencies, importance to future learning, and ways to reduce them. Swedosh ([6]) discussed the types, frequencies, and possible reasons for misconceptions exhibited by mathematics students at the University of Melbourne (U. of M.) and LaTrobe University (LaTrobe).

Success at a particular level depends heavily on previous mastery of basic concepts and also on being able to confidently use certain skills ([6]). The preparedness of students to study tertiary mathematics is determined, largely, by the level of understanding of some basic mathematical concepts which are acquired in secondary school ([6]).

Government bodies have recognised the need for a sound preparation for further studies ([7]; [8]; [9]). In A National Statement on Mathematics for Australian Schools ([9]), a major goal is that "as a result of learning mathematics in school, all students should possess sufficient command of mathematical expressions, representations and technology to continue to learn mathematics independently and collaboratively" (p. 18).

Blyth and Calegari ([10]) stated that "contrary to a widely held belief, 90% of all HSC students do apply for tertiary entrance. It is reasonable to infer from this that most students see HSC as a preparation for tertiary studies" (p.312), and, "statistics collected by the Mathematical Association of Victoria show that 75% of all tertiary courses require a pass in HSC mathematics" (p. 312). These statements confirm that students' preparedness to undertake tertiary mathematics is a significant issue and though they were made some years ago, there is plenty of anecdotal evidence to suggest that this situation has not changed dramatically.

Various attempts have been made to eliminate misconceptions. Several authors have found the "conflict teaching approach", based on Piaget's notion of cognitive conflict, to be effective in successfully resolving misconceptions relating to aspects of mathematics and physics ([13]; [14];

[15]; [16]). Inconsistencies in the learner's thinking are discussed so that the learner realises that the conceptions exhibited were inadequate or faulty and needed modification ([11]). Vinner ([12]) supports this premise and states that "there is no doubt that if inconsistencies in the students' thinking are drawn to their attention, it will help some of them to resolve some inconsistencies in a desirable way" (p. 97).

Swedosh and Clark ([16]) found that the strategy greatly reduced the frequency of mathematical misconceptions in a first year university mathematics class which comprised very bright and mathematically strong students (as indicated by their Year 12 scores).

> It is clear that by first challenging or undermining the misconception held by the students by showing the ridiculous outcomes which can flow from such 'rules', and then replacing the 'damaged' concept with the correct one, mathematical misconceptions can be, to a great extent, eliminated (p. 498).

The questions which remained were whether the improvement would persist, and whether the method would be effective with less able students. On the first of these questions:

> The results show that while a small proportion of the improvement diminished, a large improvement was still evident one year later and most of the benefit to students had been retained ([17], p. 595).

The aim in this study is to clarify whether the effect of the conflict teaching approach is generalisable in terms of students of different ability levels. Having established that this strategy is extremely effective with very able students, and that most of the benefit persists over time, it is important to learn whether the strategy is successful with average students.

Methodology

An experiment was devised in which there was a treatment group who were to be subjected to the conflict teaching approach and a control group who were not. Both groups were comprised of students who, in Semester 1, 1998, were studying Introductory Mathematics which is the easiest first year mathematics subject offered at the U. of M. The lecturers of both groups had consistently received high ratings for their teaching from their students.

To avoid distortion by inherent differences between the groups, it was important that students' backgrounds be comparable. The students reported on herein, sat both tests, had completed Mathematics Methods 3/4 but not Specialist Mathematics as part of their Victorian Certificate of Education in 1997 and enrolled at the U. of M. in 1998. There were 90 students in this category in the treatment group and 50 in the control group. Students who did not fit this category have been excluded from this study. An analysis was performed on the Year 12 results of the two groups and there was little difference.

Both groups sat the same tests at the same times. The objective was to control extraneous influences so that the effect of the teacher (as much as is practicable), the effect of being taught the material, and the effect of the testing were the same for each group, so that if a significant difference was found between the performances of students in the two groups, any difference in the improvement of the two groups could be attributed to the treatment.

The nine questions on the test were a subset of the seventeen questions used in Swedosh and Clark ([16]). These questions were at an appropriate level for these students. The earlier seventeen questions were similar to those in earlier tests at the U. of M. and at LaTrobe ([18])

which had previously had a high frequency of misconceptions exhibited by students ([6]). Some questions were similar to those provided in 'Algebraic Atrocities' ([4], p. 41). The test questions were designed so that if a student had a particular misconception, this fact would be apparent when the response of the student to that question was considered.

Both groups sat Test 1 in mid-March. Students were given ten minutes to complete the test which was ample time and did not impinge too greatly on the lecturer's time. Every test was examined and a record was kept of how many students attempted each question, answered correctly, exhibited a misconception, and had given another wrong answer.

The concepts used in Questions 2 to 7 were met incidentally by both classes but neither came across anything relevant to Questions 1, 8 or 9. Questions 2 to 7 therefore become the focus of this study. For these questions, both groups were taught the necessary concepts as they arose. Additionally, the treatment group was taught using the conflict teaching approach and was shown the absurdities which would arise if the 'rule' was used incorrectly. The fact that the approach was integrated into the regular teaching programme and was taught incrementally, avoids the Hawthorne effect and is educationally sound. The misconceptions exhibited were specifically targeted for treatment. The rationale is that having seen that the concept, when used in that way, leads to a silly conclusion, the student will avoid that misconception and be more likely to use the correct concept. Neither group met concepts relating to Questions 1, 8 or 9 and so these served as another control. It is possible that exposure to other mathematics could indirectly cause some improvement, but the absence of instruction should mean that any improvement of the groups should be comparable.

About nine weeks after Test 1, students were tested again using the same test as previously. Students were unaware that there would be a second test and therefore would not attempt to prepare for it which would potentially affect the results of Test 2. Again the tests were examined and a record kept. A statistical comparison was then able to be made between the frequency of the misconceptions on each question in Test 1 and Test 2 for each group.

After the second test, the two students with the greatest improvement were interviewed about their performance. This will be discussed in the results section of this paper.

The Test

The nine questions in the test are shown below. The most common misconception(s) are shown to the right of each question :

Simplify expressions 1-3 as fully as possible.	
1. $\frac{100!}{98!}$	$2!$
2. $3^x \times 3^x$	9^{2x}; 3^{x^2}
3. $2^x + 2^x$	4^x
Solve for x equations 4-7:	
4. $x^2 = 81$	$x = 9$
5. $x^2 - 4x = 0$	$x = 4$
6. $x^2 = x$	$x = 1$
7. $\frac{1}{x} - \frac{1}{b} = \frac{1}{a}$	$x = a + b$
8. Solve for x : $2x + 4 < 5x + 10$	$x < -2$; $x > 2$
9. Factorise $(2x + y)^2 - x^2$	$3x^2 + 4xy + y^2$

The Teaching Strategy

The conflict teaching approach was used on the treatment group for concepts required in the focus questions. By demonstrating that the misconception led to a ridiculous conclusion, it was anticipated (based on [16]) that students having that conception would discard it and replace it with the correct concept thereby reducing the frequency of misconceptions.

Most examples used to show an absurdity were numerical for ease of demonstrating the inconsistency. Examples differed from the test (numbers changed slightly) so that students would not be able to answer later by remembering. Some examples used to demonstrate either the correct concept or the absurdity caused by the damaged concept are shown below.

$$2^3 \times 2^3 = 8 \times 8 = 64 = 2^6 = 4^3 \; ; \quad 2^3 \times 2^3 \neq 2^9 \; ; \quad 2^3 \times 2^3 \neq 4^6 \; ; \quad 2^3 \times 2^3 \neq 4^9$$
$$2^3 + 2^3 = 8 + 8 = 16 \; ; \quad 4^3 = 4 \times 4 \times 4 = 64 \neq 16$$
$$x^2 = 16, \; x^2 - 16 = 0, \; (x-4)(x+4) = 0$$
$$2 + 3 = 5 \; ; \tfrac{1}{2} + \tfrac{1}{3} \neq \tfrac{1}{5} \text{ or } \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2} \; ; 4 + 4 \neq 2$$

Results

To determine whether the proportion of misconceptions made by the treatment group and the control group were significantly different, 95% confidence intervals were calculated for the differences in the population proportions of misconceptions between the two groups for the focus questions. If this interval includes zero, the difference is not significant. The formula used is

$$p_1 - p_2 \pm 1.96\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

where p_i is the proportion of misconceptions exhibited by group i and n_i is the number of subjects in group i. This test was also used on the three 'non-focus' questions, and no differences were significant.

The results from Test 1 were very similar: an average of 1.68 correct for the control group compared with 1.72 for the treatment group, 3.02 misconceptions compared with 3.11, and so on. No differences were significant for the total or for any question. Interestingly, the average frequency of 'other wrong answers' (not specifically targeted) was similar for both groups and showed little change from before treatment to after for most questions.

The results of Test 2 show a significant difference between the two groups for the overall proportions of misconceptions exhibited. The proportion for the control group dropped from 54.3% to 48.4% from Test 1 to Test 2 whereas the treatment group dropped from 55.3% to 31.3%. The proportion of correct answers given by the treatment group relative to the control group shows a corresponding increase. All questions except Question 6 show an appreciable difference and the differences were significant for Questions 2, 3 and 5.

The two students from the treatment group who had made the greatest improvement from the first test to the second were invited to an interview to discuss their performance. They were interviewed separately and both offered similar explanations for their improvement. Each stated, without prompting, that there were occasions in the second test when they were about to write an answer, but realised that using the method which they had would lead to a silly answer. Both mentioned that they had come to this decision by remembering what they had been shown in class in terms of the 'silliness' which could result from some strategies, and by substituting in numbers to see whether the answer was reasonable. The need for an alternate method was then considered and they remembered the correct one.

Conclusion

The aim of this study was to attempt to diminish the level of diversity of students by reducing the frequency of misconceptions exhibited by average students using a teaching strategy based on the notion of cognitive conflict. This strategy had been found to be effective with able students and the benefit had persisted. Clearly, this strategy would be far more useful if it could be established that it can be successfully employed when dealing with students of diverse ability and background levels.

The results of the experiment provide strong evidence that the conflict teaching strategy can be successfully employed to significantly decrease the frequency of misconceptions exhibited by average students. The misconceptions specifically targeted were decreased for the treatment group by a significantly greater amount than for the control group, whereas the 'other wrong answers' (which were not specifically targeted) were not, and neither was the frequency of misconceptions on the three 'non-focus' questions. Incorporating this strategy into one's teaching has major benefits in that it is extremely simple to implement, it is likely that fewer students will have these misconceptions, and, as a result, many will directly improve their chances of being successful in their future mathematical studies. The use of this strategy, especially with regard to the concepts herein, is particularly applicable to middle and senior secondary mathematics as well as to beginning tertiary mathematics.

Acknowledgments

I would like to acknowledge the cooperation of Frank Barrington and Angie Byrne and the advice provided by David Clarke. Their efforts are sincerely appreciated.

References

[1] Bell, A.W., (1982). Diagnosing students misconceptions. *The Australian Mathematics Teacher*, 38(1), 6-10.

[2] Davis, R.B., (1984). *Learning Mathematics : The cognitive science approach to mathematics education*, London : Croom Helm.

[3] Farrell, M.A. (1992). Implementing the professional standards for teaching mathematics: Learning from your students, *Mathematics Teacher*, 85(8), 656-659.

[4] Margulies, S. (1993). Tips for beginners : Algebraic atrocities. *Mathematics Teacher*, 86(1), 40-41.

[5] Perso, T. (1992). Making the most of errors, *The Australian Mathematics Teacher*, 48(2), 12-14.

[6] Swedosh, P. (1996). Mathematical misconceptions commonly exhibited by entering tertiary mathematics students. In P. Clarkson (Ed.), *Technology in Mathematics Education*, Mathematics Education Research Group of Australasia. Melbourne, 534-541.

[7] Ministry of Education (Victoria), (1984). *Ministerial Paper Number 6 – Curriculum development and planning in Victoria*, Government Printer, Melbourne.

[8] Victorian Government, (1987). *Victoria The Next Decade*, Government Printer, Melbourne.

[9] Australian Education Council, (1990). *A National Statement on Mathematics for Australian Schools*, Curriculum Corporation, Carlton, Victoria.

[10] Blyth, B., & Calegari, J. (1985). The Blackburn Report: Nightmare or Challenge? From a Tertiary Perspective. In P. Sullivan (Ed.), *Mathematics – Curriculum, Teaching & Learning*, The Mathematical Association of Victoria, Melbourne, 309-314.

[11] Tirosh, D. (1990). Inconsistencies in students mathematical constructs, *Focus on Learning Problems in Mathematics*, 12,(3-4), 111-129.

[12] Vinner, S. (1990). Inconsistencies: Their causes and function in learning mathematics, *Focus on Learning Problems in Mathematics*, 12,(3-4), 85-98.

[13] Stavy, R., & Berkovitz, B., (1980). Cognitive conflict as a basis for teaching quantitative aspects of the concept of temperature, *Science Education*, 64, 672-692.

[14] Strauss, S. (1972). Inducing cognitive development and learning: A review of short-term training experiments: 1) The organismic-developmental approach, *Cognition*, 1, 329-357.

[15] Swan, M. (1983). *Teaching decimal place value. A comparative study of "conflict" and "positive only" approaches*, University of Nottingham, Shell Centre for Mathematical Education, Nottingham, England.

[16] Swedosh, P., & Clark, J. (1997). Mathematical Misconceptions – Can We Eliminate Them? In F.Biddulf & K. Carr (eds.), *People in Mathematics Education*, (2), Mathematics Education Research Group of Australasia, Waikato, 492-499.